MEMORANDUM

To: Stan Carpenter

From: William Keyes

Date: 13 September 1965

Subject: HYPERGEOMETRIC DISTRIBUTION:

1. EXPONENTIAL FAILURE LAW:

To approximate the actual failure distribution, the exponential failure law is assumed. Let F denote the number of components which would fail when N components of MTTF = M hours are tested for t hours each. Then

$$F = N\left(1 - e^{-\frac{t}{M}}\right) \quad \ldots\ldots\ldots \quad (1)$$

11. HYPERGEOMETRIC DISTRIBUTION:

Given a lot of N gyros of which F (determined from equa. (1)) are defective, what is the probability of finding i defectives in a random sample of n units?

Let X = hypergeometric random variable, the number of defectives drawn from a sample of n gyros which were drawn from a population of N gyros, (11/8/66) of which F are defective.

$$P(X = i) = \frac{\text{No. of favorable events}}{\text{No. of possible events}}$$

$$= \frac{\text{No. of ways to get i defectives and(n-i) non-defectives}}{\text{No. of ways to select n samples from lot size N}}$$

Numerator = (# ways to select i defectives from F defectives)

X (# ways to select (n-i) non-defectives from original (N-F) lot of non-defectives)

$$= \binom{F}{i}\binom{N-F}{n-i} \quad \text{where} \quad \binom{F}{i} = \frac{F!}{i!\,(F-i)!} \quad \text{etc.}$$

Denominator = # Combinations of N objects taken n at a time.

$$= \binom{N}{n}$$

Let p (i ; N, F, n) = probability of finding i defectives in a sample of n gyros drawn at random from a lot of N gyros of which F are defective.

$$\text{Then } p\,(i;\,N,\,F,\,n) = \frac{\binom{F}{i}\binom{N-F}{n-i}}{\binom{N}{n}} \quad \ldots\ldots (2)$$

The sum of hypergeometric probabilities must be 1

$$\sum_{i=o}^{N} \frac{\binom{F}{i}\binom{N-F}{n-i}}{\binom{N}{n}} = 1$$

$$\text{defining } \binom{N}{n} = o \text{ when } r > N$$

With an acceptance number of failures C, a partial sum determines confidence factor:

$$\sum_{i=o}^{C} p\,(i;\,N,\,F,\,n) = \text{sum of probabilities of finding } i = o,\ 1,\ \ldots\ C \text{ defectives}$$

Define confidence factor P

$$\sum_{i=o}^{C} p\,(i;\,N,\,F,\,n) = \sum_{i=o}^{C} \frac{\binom{F}{i}\binom{N-F}{n-i}}{\binom{N}{n}} \equiv 1 - P\ \ldots\ (3)$$

To establish an acceptance test which will project a MTTF of $M = rt$ (t = testing time) with confidence P from a lot of N units with acceptance number C, the number of samples, n, which are needed is found:

a. Calculate F from eq. (1) to nearest integer

b. Calculate n from eq. (3)

If n units from a lot of N gyros are tested for t hours each and C random failures are observed, the MTTF with confidence P is calculated:

a. Solve equa.(3) for F

b. Solve equa. (1) for M

EXAMPLE:

In a procurement of 100 units, the MTTF is required to be 100, 000 hours with 80% confidence and acceptance no. of failures C = 1. Find minimum sample size for 10, 000 hours of testing.

a. $F = N\left(1 - e^{-\frac{t}{M}}\right) = 100\left(1 - e^{-0.1}\right) = 10$

b. $$\frac{\binom{F}{o}\binom{N-F}{n} + \binom{F}{1}\binom{N-F}{n-1}}{\binom{N}{n}} \leq 1 - P$$

try n = 20

$$\frac{\binom{90}{20} + 10\binom{90}{19}}{\binom{100}{20}} = .42$$ which is $>$ than the acceptable "risk" of 1-P=.20

$\therefore$ 20 samples is too few

try n = 30

$$\frac{\binom{90}{30} + 10 \times \binom{90}{29}}{\binom{100}{30}} = .194$$ which is within the acceptable "risk" of .20

Therefore if 30 units are tested for 10,000 hours each and 1 random failure is observed, the MTTF of each of the original lot of 100 units is 100,000 hours with 80% confidence.

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